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PRESSURE GENERATION DURING THE DRYING OF A POROUS HALF-SPACE

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Abstract – A simple model of the drying of a porous media is considered. An analytic solution for the drying of a porous half-space is presented and maximum pressures are calculated.

NOMENCLATURE

k, permeability;
K, thermal conductivity;
L, latent heat;
$P(x,t)$, pressure;
P_A, atmospheric pressure;
R_0, gas constant;
t, time;
$T(x,t)$, temperature;
T_E, evaporation temperature;
T_o, outside temperature;
T_s, initial temperature;
$V(x,t)$, velocity;
x, coordinate;
$X(t)$, position of evaporation front.

Greek symbols

$\rho(x,t)$, moisture density;
ρ_w, density of water;
κ, thermal diffusivity;
ϕ, porosity;
Θ, dimensionless temperature;
η, similarity variable;
$\lambda, \beta, \varepsilon, \alpha, \nu, \bar{\gamma}$, dimensionless parameters.

1. INTRODUCTION

A COMPREHENSIVE mathematical description of the drying of porous media has been developed by Luikov (cf. [1] a recent review). The system, which consists of a number of parabolic partial differential equations which describe the temperature, moisture and pressure distributions, is very complex. In fact to employ the full Luikov system a large number of parameters are required, and in practice, this is not often possible to achieve. Thus, more recently workers [2–6] have been looking at restricted classes of the system. Essentially, this restriction has involved the assumption that the pressure does not significantly influence the temperature or moisture distribution during drying.

The system of equations describing the processes of simultaneous heat and mass transfer only has been examined in detail by Gupta [2], Cho [3,4] and Mikhailov [5]. Gupta [2] considered a further restricted case ignoring the Soret effect (i.e. assuming the Posnov number is small which is equivalent to neglecting the moisture diffusion due to temperature variation). His approach involved the introduction of a moving evaporation front which was then tracked numerically through a half-space. Cho [3] subsequently developed an analytic solution to this problem and more recently both he [4] and Mikhailov [5] have produced solutions when the Soret effect is included.

The problem to be described here is not so much concerned with the simultaneous heat and mass transfer as with the pressures generated during the drying of a porous medium. This work was stimulated by a study of iron ore pellets during their drying [6]. Before drying commences the pellets consist of a large number of small particles bonded together by a variety of mechanisms [7], plus a relatively high proportion of voids ($\sim 30\%$) saturated with moisture. On economic grounds it is highly desirable to complete the drying process as quickly as possible. However, if the drying rate is too fast high pressure gradients may be generated within the pellets which may cause them to break up – a highly uneconomic feature of operation. In an attempt to identify the factors which are most likely to affect the pressures generated during drying, a simple mathematical description was formulated [6]. The resulting system of equations had no analytic solution. Hence, in order to lend credence to the computed results an analytic solution to a simpler problem was developed.

In this paper, the solution to that simplified problem is described; that is, an analytic solution for the pressure generated during the drying of a porous half space is developed. It is probably worth noting at this stage that the approach to the calculation of the pressure distribution is somewhat simplified when compared to that of Luikov [1]. However, the main advantage of this method is that it requires very few parameters (by comparison to Luikov) and produces results which appear to be realistic [6].

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2. THE MATHEMATICAL ANALYSIS

Basically, the analysis presented here attempts to provide an approximate description of the following aspects of the drying process: (i) the conduction of heat through the medium; (ii) the evaporation of moisture within the medium; and (iii) the pressure generated by the flow of moisture vapour through the medium.

2.1. *Assumptions*

In the analysis of the iron ore pellet it has been experimentally validated that the drying rate was high compared to the rate of moisture diffusion [6]. This is the main assumption again utilised in the analysis described here. Thus, in the formulation of the analysis the following assumptions are implicit: (i) the temperature and moisture distributions are independent of pressure; (ii) heat flows by conduction only through the medium; (iii) the half-space is effectively divided into two regions (a) containing moisture only (at a constant level), and (b) containing moisture vapour only diffusing out; (iv) the vapour does not take part in the heat-transfer process; (v) the vapour flow is determined by the Darcy and ideal gas laws, and the temperature of the vapour is the associated solid temperature; (vi) the basic interaction between the pressure and the temperature and moisture is via the vapour generation at the moving front.

2.2. *Evaluation of the temperature and moisture distributions*

A semi-infinite porous slab ($x \geqslant 0$) porosity ϕ, initial temperature T_s and saturated with moisture throughout is considered. The temperature, $T(x, t)$, is governed by the equation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} \quad \begin{array}{l} 0 < x < X(t) \\ X(t) < x < \infty \end{array} \tag{1}$$

subject to the boundary conditions

$$T(0, t) = T_o \tag{2a}$$

$$T[X(t), t] = T_E \tag{2b}$$

$$T(x, 0) = T_s \text{ for all } x \tag{2c}$$

where $X(t)$ is the position of the evaporation front at time t, T_o is the outside temperature, T_E is the evaporation temperature ($T_s < T_E < T_o$) and κ is the thermal diffusivity. For the sake of simplicity κ is assumed to take the same value in both regions although the analysis could be extended to take account of varying thermal properties (see, for example, Cho [4]).

Across the evaporation front the heat and mass balance gives

$$\phi \rho_w L \dot{X} = -\left\{ K_1 \frac{\partial T}{\partial x}\bigg|_{x-} - K_2 \frac{\partial T}{\partial x}\bigg|_{x+} \right\} \tag{3}$$

where ρ_w is the moisture density, L is the latent heat, $K_{1,2}$ are the thermal conductivities of the porous medium in its dry and saturated state respectively and ϕ is the material porosity.

These equations really form a simplified version of Cho's analysis [3], representing a very simple Stefan problem. This particular problem has a Neumann type solution [8]

$$T(x, t) = T_o - (T_o - T_E)\theta(\eta) \tag{4}$$

with the position of the evaporation front being given by

$$X(t) = 2\lambda(\kappa t)^{1/2}. \tag{5}$$

In the above equations

$$\eta = \frac{x}{2(\kappa t)^{1/2}} \tag{6}$$

$$\theta(\eta) = \begin{cases} \dfrac{erf(\eta)}{erf(\lambda)} & (\eta < \lambda) \\ \beta - (\beta - 1)\dfrac{\operatorname{erfc}(\eta)}{\operatorname{erfc}(\lambda)} & (\eta > \lambda) \end{cases} \tag{7}$$

$$\beta = \frac{T_o - T_s}{T_o - T_E} \tag{8}$$

and λ is the solution of the transcendental equation

$$\lambda = \frac{\varepsilon \, e^{-\lambda^2}}{(\pi)^{1/2}} \left\{ \frac{1}{\operatorname{erf}(\lambda)} - \frac{K_2}{K_1} \frac{(\beta - 1)}{\operatorname{erfc}(\lambda)} \right\} \tag{9}$$

where

$$\varepsilon = \frac{K_1(T_o - T_E)}{\phi \rho_w L \kappa}. \tag{10}$$

Equations (4)–(10) define the simultaneous heat- and mass-transfer drying process under the assumptions described in Section 2.1. Equation (4) describes the temperature distribution; the moisture distribution is a simple step function with the step defined by equation (5). A table of solutions of equation (9) is given in Carslaw and Jaeger [8].

2.3. *Evaluation of the pressure distribution*

The gas flow velocity, $V(x, t)$ and pressure, $P(x, t)$ may be related by Darcy's law [9]

$$v = -\frac{k}{\mu}\frac{\partial P}{\partial x} \quad 0 < x < X(t) \tag{11}$$

and the continuity equation

$$\frac{\partial}{\partial x}(\rho v) = -\phi \frac{\partial \rho}{\partial t} \tag{12}$$

where k is the medium permeability, μ is the viscosity and ρ is the moisture density. Equation (12) includes pressure by virtue of the ideal gas law

$$P = \rho R_0 T \tag{13}$$

where R_0 is the gas constant for water vapour.

On eliminating v from equations (11) and (12) the pressure in the dry part of the slab [$0 < x < X(t)$] is governed by

$$\frac{\partial}{\partial x}\left(\rho \frac{\partial P}{\partial x}\right) = \frac{\phi \mu}{k}\frac{\partial \rho}{\partial t} \tag{14}$$

subject to the boundary conditions

$$P = P_A \quad \text{at} \quad x = 0 \tag{15}$$

(where P_A is the atmospheric pressure) and

$$\rho_w \dot{X} = \frac{\rho k}{\mu}\frac{\partial P}{\partial x} \quad \text{at} \quad x = X(t) \tag{16}$$

which arises as a consequence of continuity at the moving front.

By non-dimensionalising and introducing the similarity variable, η, the equations (14)–(16) reduce to a set of ordinary non-linear differential equations, i.e.

$$\frac{d}{d\eta}\left(\Delta \frac{dp}{d\eta}\right) = -\alpha\eta \frac{d\Delta}{d\eta} \tag{17}$$

with

$$p = 1 \quad \text{at} \quad \eta = 0 \tag{18}$$

and

$$\Delta \frac{dp}{d\eta} = \lambda v \quad \text{at} \quad \eta = \lambda \tag{19}$$

where

$$p = \Delta[1 - \bar{\gamma}\theta(\eta)] \tag{20a}$$

$$p = P/P_A \tag{20b}$$

$$\Delta = \rho R_0 T_o / P_A \tag{20c}$$

$$\alpha = 2\frac{\mu\phi\kappa}{kP_A} \tag{20d}$$

$$\bar{\gamma} = (T_o - T_E)/T_o \tag{20e}$$

and

$$v = 2\frac{\rho_w R_0 T_o \mu\kappa}{kP_A^2}. \tag{20f}$$

By assuming that α is small [e.g. for iron ore pellets $\alpha \sim O(10^{-5})$], equations (17) and (19) reduce to

$$\Delta \frac{dp}{d\eta} = \lambda v \ \text{for all} \ 0 < \eta < \lambda. \tag{21}$$

This integrates exactly to give

$$p^2 = 1 + 2\lambda v\left\{\eta - \frac{\bar{\gamma}}{\text{erf}(\lambda)}\left[\eta\,\text{erf}(\eta) - \frac{1}{(\pi)^{1/2}}(1 - e^{-\eta^2})\right]\right\}. \tag{22}$$

Examination of equation (22) shows that the maximum pressure, $p$max will always occur at the evaporation front (i.e. where $\eta = \lambda$). A typical set of results is shown in Fig. 1 where the (dimensionless) maximum pressure is plotted against v for various values of ε. Since both v and ε are proportional to the ambient temperature, ε is inversely proportional to the material porosity, and v is inversely proportional to the permeability, the results indicate that the maximum generated pressure increases with the ambient temperature and decreases with material porosity or permeability.

For many situations of practical importance ε is small (see, for example, [6], or in the drying of refractory shapes where $T_o - T_E \simeq T_s \simeq 20°C$). In such cases equation (9) may be simplified to $\varepsilon = 2\lambda^2$, in which case, we may approximate the maximum generated pressure by

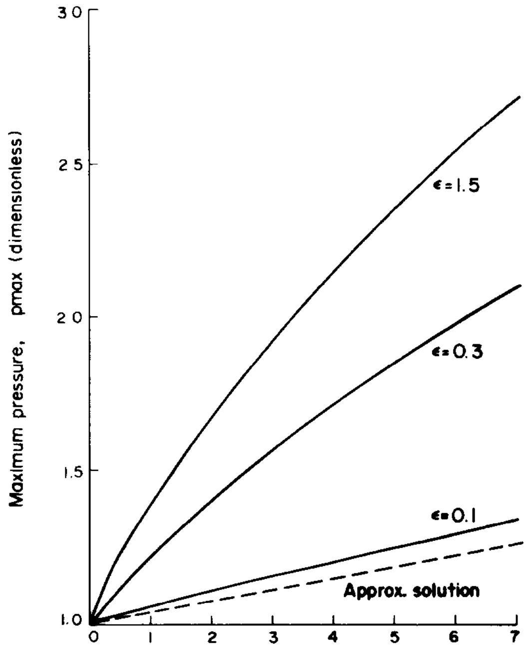

FIG. 1.

$$p^2 \max = 1 + \frac{R_0(T_o^2 - T_E^2)\mu K_1}{\phi L P_A^2 k} \tag{23}$$

$$= 1 + \frac{\varepsilon v}{2}(2 - \bar{\gamma}).$$

The dashed line Fig. 1 shows the comparison of the approximate formulation to the exact result for $\varepsilon = 0.1$. Equation (23) thus provides a simple useful equation to describe the maximum generated pressure which, by virtue of substitutions from equations (8), (9), (10) and (20), includes the effects of variations in: (i) the ambient and initial temperatures; (ii) the thermal properties of both the material and the fluid; plus (iii) the porosity and permeability of the porous medium. Finally, note that equation (23) shows explicitly the dependence of the maximum generated pressure on both the porosity and permeability.

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GENERATION DE PRESSION PENDANT LE SECHAGE D'UN CORPS POREUX SEMI-INFINI

Résumé—On considère un modèle simple de séchage d'un milieu poreux. On présente une solution analytique du séchage d'un corps poreux semi-infini et les pressions maximales sont calculées.

DRUCKVERLAUF WÄHREND DES TROCKNUNGSVORGANGES EINES PORÖSEN HALBRAUMES

Zusammenfassung—Ein einfaches Modell des Trocknungsvorganges eines porösen Mediums wird betrachtet. Eine analytische Lösung für das Trocknen eines porösen Halbraumes wird angegeben, und die maximalen Drücke werden berechnet.

РОСТ ДАВЛЕНИЯ ПРИ СУШКЕ ПОРИСТОГО ПОЛУОГРАНИЧЕННОГО ТЕЛА

Аннотация — Рассматривается простая модель сушки пористых сред. Представлено аналитическое решение для сушки пористого полуограниченного тела и рассчитаны значения максимального давления.